Physical sense of renormalizability

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Abstract

A plausible physical interpretation of the renormalizability condition is given. It is shown that renormalizable quantum field theories describe such systems wherein the tendency to collapse associated with vacuum fluctuations of attractive forces is suppressed by vacuum fluctuations of kinetic energy. Relaying on the classification of topological types of evolution of point particles and analysing the problem of the fall to the centre, we obtain a general criterion for preventability of collapse which states that the spectrum of the Hamiltonian must be bounded from below. The holographic principle is used to explain the origin of anomalies and make precise the relation between the renormalizability and the reversibility.

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1 Introduction

A stirring event in the modern history of high energy physics was the awarding of the 1999 Nobel Prize in physics to Gerardus 't Hooft and Martinus Veltman "for elucidating the quantum structure of electroweak interactions in physics". Experts are well aware of their particular credit for the proof of the renormalizability of the Yang–Mills–Higgs model which underlies this official formulation of the Royal Swedish Academy of Sciences. As for the general physical audience, the importance of this advance is unlikely to be fully realizable because the physical sense of the renormalizability has been only slightly or not at all clarified in the literature. It is felt, there comes a time where the discussion of the renormalizability on a level close to intuitive is quite relevant.

One should distinctly discriminate between the notion of the *renormalization* and that of the *renormalizability*. Our aim is to take a close look at the latter; the physical sense of the former is sufficiently clear, so we restrict ourselves to the following brief remark.

It was known even in the 19th century that the renormalization of mass is to be expected in systems with infinite degrees of freedom. Let us imagine a spherical body of mass M_0 moving with speed $\mathbf{v}$ through a fluid. The hydrodynamics states that the kinetic energy of the system composed of this moving body and a portion of the fluid dragged by it is $\frac{1}{2}\,M\mathbf{v}^2$ where $M = M_0 + \delta M$, with δM being the apparent additional mass equal to half the mass of the fluid displaced by the body, and, besides, a force applied to the body produces the acceleration inversely related to M. Thus dynamical laws in fluids remain identical to those in vacuum, but the mass appearing in these laws can vary considerably. For example, let a ping-pong ball be placed in a vessel filled with mercury, then we are dealing with an object which is 80 times as massive as that is in the habit. If the body cannot be withdrawn from the fluid, its inertia is specified by the renormalized mass M, and M_0 cannot be measured. Reasoning from the analogy between the hydrodynamical medium and the ether, J. J. Thomson introduced the electromagnetic mass of a charged particle δm which must be added to its mechanical mass m_0 to give the observable mass m. (For further elaborations of this idea by Thomson, Lorentz and Kramers see [1].)

In the relativistic quantum domain, we encounter processes of creations and annihilations of particles. They give rise to a specific quantum field phenomenon, the *vacuum polarization*, which is responsible for the coupling constant renormalization. Indeed, let some electron be localized within a region of size less than half its Compton wavelength. Then, due to the Heisenberg uncertainty principle, this will result in

the energy fluctuation sufficient for the creation of a virtual pair of an electron and a positron. The less the localization region, the greater are the energy fluctuations, and hence the greater is the number of pairs that might be created and annihilated here. The electron attracts virtual positrons and repulses virtual electrons, thus dipoles of the separated pairs screen its charge. The electron charge e measured at a distance greater than its Compton wavelength is renormalized relative to its bare charge e_0 owing to this screening. Furthermore, the mass m of the electron wrapped up in the coat of virtual pairs turns out to be renormalized relative to its bare mass m_0.

Thus the vacuum of the quantum field theory (QFT) plays the role of a medium that renormalizes masses and coupling constants. The trouble, however, is that such renormalizations are infinite for physically interesting Lagrangians, or, more technically, the calculation of corrections to masses and coupling constants following the Feynman rules is confronted by ultraviolet divergences.

Mathematically, the presence of these divergences is due to the fact, first established by N. N. Bogoliubov [3], that the multiplication of distributions is ill defined. The renormalization theory enabling the absorption of ultraviolet divergences by infinite renormalizations of masses and coupling constants gives an unambiguous prescription for the definition of the product of propagators at points where their arguments coincide. But this prescription is far from all-inclusive. According to the renormalization theory, all the local quantum field theories can be separated into two classes, renormalizable and nonrenormalizable. Take for example a system specified by fields with spins 0 and 1/2[1]. Let the Lagrangian of interaction $\mathcal{L}_I$ be a polynomial in fields, with the monomial of the power i containing a product of b_i scalar and f_i Dirac fields, and k_i derivatives. The rule of the renormalizability by the power-counting, or, more precisely, by the index

$$\omega_i = b_i + \tfrac{3}{2}\, f_i + k_i - 4 \tag{1}$$

reads: The theory is *superrenormalizable* when $\omega_i < 0$ for all i, *renormalizable* when $\omega_i \leq 0$, and *nonrenormalizable* when $\omega_i > 0$ if only for a single i[2].

At present the renormalization procedure, known in the mathematical literature as the R-operation, is developed in every respect with a suitable rigor (the way of its development and current status are reviewed in [4]). There are textbooks where this procedure is presented rather skilfully, e. g., [5]–[8]. The study of its applied aspects, in particular the calculation of multiloop diagrams in renormalizable theories, continues on its way [4, 9]. The mathematical nature of the R-operation gradually deepens; it became clear recently that it is a special instance of a general mathematical procedure of multiplicative extraction of finite values based on the Riemann–Hilbert problem [10].

As to the notion of the renormalizability, it went a long but still incomplete way of development (impressed in excellent surveys, e.g., [11]–[14], and books [2, 15]). Referring the reader for details and the bibliography to these and quoted below works, we recall only some facts directly related to our subject. Of course, our short excursion into the history of the renormalizability is on no account intended to the role of an express-analysis of key events in the QFT during the last 50 years, and mentioning selected names and papers does not imply our priority preference or evaluation of importance of any advances. We would like only to clear up motivations of 'old' and 'new' activities that, in one way or another, have a bearing on the renormalizability.

In 1949, F. Dyson [16] showed that the renormalization of masses and charges is sufficient for the removal of ultraviolet divergences in quantum electrodynamics (QED). He observed also that it is possible to absorb all the infinities by a redefinition of a finite number of parameters in the Lagrangian only for a certain kind of theories that he called renormalizable. Since then the renormalizability became a criterion for the theory selection. In the 1970s, this criterion was slightly revised: A theory is taken to be consistent when it is not only renormalizable by the power-counting rule but also free of anomalies of local symmetries (even though anomalies of global symmetries are harmless and even desirable). The divergence phobia was replaced by the interest in such an infrequent occurrence when they can be removed: "The divergences of quantum field theory must not be viewed as unmitigated defects; on the contrary, they convey crucially important information about the physical situation, without which most of our

[1]Throughout this paper we choose, unless otherwise indicated, units such that $\hbar = c = 1$.

[2]This rule can be reformulated as follows: The theory is renormalizable when the field dimensionality of $\mathcal{L}_I$ expressed through the dimensionality of mass μ is $\mu^{4+\omega}$ with $\omega \leq 0$, and hence every coupling constant is either dimensionless or of dimension of mass to a positive power.

theories would not be physically acceptable [...] One cannot escape the conclusion that Nature makes use of anomalous symmetry breaking, which occurs in local field theory owing to underlying infinities in the mathematical description” [17].

We draw attention to the instrumental character of the criterion. It reflects the view on the acceptable QFT as a collection of perturbation rules compatible with the renormalization procedure. Nonrenormalizable theories are thought to be bad not due to the fact that they are based on some physical ‘pathologies’ but merely because we do not know how to handle them. All reverses of fortune of the criterion are related to this feature. At different times, the perception of the criterion changed drastically, from decisive rejection of all local theories, both renormalizable and nonrenormalizable, to full tolerance of every theory, nonrenormalizable including; yet one attempted seldom if ever to clarify the difference of renormalizable and nonrenormalizable theories in their physical essence.

The crisis of the 1950–1960s in QFT burst out in connection with the discovery of the ‘null-charge’ phenomenon by L. D. Landau and I. Ya. Pomeranchuk [18], and independently by E. S. Fradkin [19]. They revealed that the vacuum polarization in QED and other models is so strong that the observed coupling constants are subject to a complete screening irrespective of values of bare coupling constants. Another troublesome surprise was the discovery of the photon ‘ghost’ state [20]. Landau regarded the charge nullification as evidence of logical inconsistency of QED and the conception of local interactions as such [21]. The renormalizability principle was buried under debris of the Lagrangian formalism for a long time. “Under the influence of Landau and Pomeranchuk, a generation of physicists was forbidden to work on field theory” [13].

In the early 1970s, the ‘gauge revolution’ occured and the ‘Golden Age’ of renormalizable theories came. The breakthrough rested on the proof of the renormalizability of the Yang–Mills–Higgs model [22] and the discovery of the asymptotic freedom [23]. The renormalizable $SU(3)\times SU(2)\times U(1)$ gauge theory of strong, electromagnetic and weak interactions was established and received the name Standard Model. The idea of a confluence of running coupling constants in the vicinity of 10^{16} GeV initiated efforts to build the Grand Unification of three fundamental interaction in the framework of gauge theories with a simple group of internal symmetry, for instance $SU(5)$ or $O(10)$. All these milestones are now widely known and are expounded in textbooks.

It was anticipated that the availability of the asymptotic freedom would rehabilitate the four-dimensional (4D) quantum field theory. “One can trust renormalization theory for an asymptotically free theory, independent of the fact that perturbation theory is only an asymptotic expansion, since it gets better and better in the regime of short distances” [13]. One believed that the S matrix in such theories would be free of the Landau ghost. However, it became soon clear that the ghost simply migrated from the ultraviolet region to the infrared. As to the quantum chromodynamics, this finding implied only that the confinement is a nonperturbative effect. Nevertheless, one was forced to bid farewell to the dream that asymptotic theories are consistent already on the perturbation level.

The proximity of the Grand Unification scale 10^{16} GeV to the Planck mass $M_{\rm P}=k^{-1/2}=1.22\times10^{19}$ GeV where k is the Newton gravitation constant was an impetus to the unification of all four fundamental forces, gravity including. But a short time later, the renormalization ideology reached a deadlock for the nonrenormalizability of gravitation.

By the end of the 1970s, rapidly grew up the supergravity [24]. Due to a remarkable feature of supersymmetric Yang–Mills theories, the cancellation of divergences in the one-loop approximation (some theories prove finite in all perturbation orders), a cardinal change of attitude had been seen on what ultraviolet behavior should be demanded from consistent field theories. The requirement of renormalizability was replaced by the condition of *finiteness* [25]. Notice, we are concerned not with a cutoff at the Planck length $l_{\rm P}=1.6\times10^{-33}$ cm, but with a cancellation of divergences which must be ensured by the ‘true’ field contents. One looked to the eleven-dimensional $\mathcal{N}=1$ supergravity since 11 is both the minimal dimension for $SU(3)\times SU(2)\times U(1)$ to be the internal symmetry group and the maximal dimension compatible with the supersymmetry of fields with spin $J\le 2$. A mechanism of compactification of 7 redundant dimensions was invented in the spirit of the Kaluza–Klein ideology.

However, the project failed: The 4D on-shell supergravity turns out to be finite only up to the two-loop approximation while the 11D supergravity suffers even from one-loop divergences. Furthermore, any odd-dimensional theory cannot be chiral, and the compactification gave no way of deriving a chiral 4D theory describing the observed world with the inherent asymmetry between the right and the left from

the 11D supergravity.

Theorists fastened their eyes on superstrings [26]–[30]. The quantum string theory is in general free of ultraviolet divergences but is plagued by anomalies. Five superstrings was shown to be free of anomalies: The open string with the $\mathcal{N}=1$ supersymmetry and the $SO(32)$ gauge symmetry, the type I string, two closed strings with the $\mathcal{N}=2$ supersymmetry, chiral and non-chiral, the type IIA and IIB strings, and two closed strings with different constructions of right and left sectors possessing the $E_8 \times E_8$ and $SO(32)$ gauge symmetries, the heterotic strings. A consistent quantization of superstrings is feasible only in 10 dimensions. The most promising was the heterotic $E_8 \times E_8$ string since it can incorporate the Standard Model. It remained only to compactify spacetime from 10 to 4 dimensions and reduce too high supersymmetries. The Calabi–Yau manifolds and orbifolds offered prospects for tackling these tasks. A consistent theory containing both semiclassical supergravity and Grand Unification with the chiral representation of quarks and leptons in the low energy limit eventually resulted. It was named 'Theory of Everything'.

Why strings are free of the ultraviolet diseases? The supersymmetric cancellation is 'almost immaterial'; bosonic strings are finite as well, but there is a tachyon in their spectrum which is removable with the aid of the supersymmetry. The finiteness of the string theory is often associated with the fact that strings are nonlocal. Strings are extended objects, and this should presumably provide the cutoff near the energy scale $M_{\rm P}$. But the string interaction is known to be local, for example, two open strings merge into a single one only when their ends contact. Meanwhile the cutoff due to nonlocal form-factors means that the *interaction is smeared out* over a finite region [31]. If the interaction is not smeared out, then, as was shown in [32], divergences do not disappear[3]. The reason for the good ultraviolet behavior of strings is likely to be the fact that we are dealing with a local conformal QFT on *two-dimensional* manifolds that are formed by the world sheets swept out in the course of the string evolution. In some cases this two-dimensional conformal theory is actually the only which is at our disposal. For example, in the heterotic construction, X_μ is a boson field that cannot be interpreted as the string coordinate in the target spacetime. That the finite length of strings does not assure the ultraviolet finiteness can be seen from the comparison of strings with membranes. Indeed, attempts to build an appropriate local QFT on three-dimensional world volumes of the membrane revert us to the problem of ultraviolet divergences.

So, the string theory is free of ultraviolet divergences. But in lieu of them, new conceptual difficulties arose. Most notable of them is the uniqueness problem. The Theory of Everything must be unique by its very nature. It should stand out against all possible theoretical schemes not only because of its best phenomenology reflection but also because of the lack of mathematical inconsistencies, unique to it. We have instead five consistent theories. Things get worse if we compactify 6 redundant dimensions: An abundance of the Calabi–Yau configurations was found which describe vacuum states of the same energy, and it is not clear how this collection can be narrowed.

Another way of solving this problem is to show that all these theories are the manifestation of the same physics, but in various contexts, e. g., in regimes of strong and weak coupling, i. e., every two of them are linked by some duality transformation [33]. Lately, it was understood that this is actually the case: All superstrings and soliton-like objects of the 11D supergravity (branes, black holes, etc.) are tightly entangled in a duality web as a unified scheme, that was called the M theory [34]. (A rigorous justification of the dualities would be possible if we were aware of all nonperturbative solutions of the theories under examination. Unfortunately, one failed to find exact solutions in the strong coupling regime, and, therefore, many dualities remain plausible conjectures.) A striking feature of the M theory is that it describes some 11D realm, with the chiral $E_8 \times E_8$ string emerging due to the compactification realized on a line-segment. Another property is the presence of extended objects of different dimensions capable to mutual conversions. At present, it is not clear what degrees of freedom are fundamental in M theory, in any case, those are not strings or branes (for a readable review of M theory see [35]).

This opens the door to construct the finite, free of anomalies, unique theory[4] embodying the first principles of the physical world. Is renormalizability as yet required as a fundamental principle? Maybe it is an appropriate time to say good-bye to it. One has at least two objection to this. Firstly, the M theory is a formidable project. It is unlikely that success will be achieved if one takes no lesson from pre-

[3] We will return to the discussion of this instructive result at the end of the paper.

[4] Meanwhile it should be remembered alternative approaches, e.g., the loop quantum gravity [38].

stringy efforts to cope with the treatment of systems with infinite degrees of freedom. (Were these efforts actually doomed to failure?) Secondly, as R. Jackiw [17] noted: "I wonder where within the completely finite and non-local dynamics of string theory are we to find the mechanisms for symmetry breaking that are needed in order to explain the world around us". Indeed, how can we reasonably explain the symmetry violations differently than considering them as healed up scars of ultraviolet wounds? In the flawed symmety problem, the finitism cease to be the good.

Besides, there is a more technical problem. Lowering ourselves from the Planck height into the region of experimentally attainable energies, we cannot take advantage of the finiteness of the string theory for calculating processes with usual particles, for example quarks, since we do not know the genuine mechanism of the dimension reduction from 10 to 4. For comparison recall that classical relativistic mechanics makes it possible to calculate both relativistic and non-relativistic motions; in some cases (for example, the particle motion in a plane electromagnetic wave) this calculation turns out to be even more simple than that in Newtonian mechanics. Thus the string theory gave no answer to the question put in the title of the famous Gell-Mann talk at the Shelter Island Conference in 1983 [14].

Let us turn to last pages of the history of the renormalizability (for more extended reviews see [36, 37]) and raise the naive question: Why are three fundamental interactions, strong, electromagnetic, and weak, renormalizable? It would be ridiculous to think that Nature plays at give-away with us arranging part of laws in such a way that we were able to realize them by means of perturbative calculations.

In the 1970s, approaches levelling differences between renormalizable and nonrenormalizable theories came into particular prominence. K. Wilson revived interest in the renormalization group as a useful tool in studying the situation in QFT beyond the perturbation theory (for the history and present status of the renormalization group investigations see [39]). By analogy with the critical point in phase transitions of the second kind, where fluctuations of all scales are comparable in magnitude, one can define the notion of ultraviolet fixed point, approaching to which renders quantum fields asymptotically scale invariant (for more detail see [40]). This notion is the starting point in the S. Weinberg program of 'asymptotic safety' which states that coupling parameters tend to a fixed point when the scale of their energy normalization tends to infinity [41]. For the Gaussian fixed points, the asymptotic safety is equivalent to the renormalizability in the usual sense (with the null-charge problem being conclusively avoided), while, in the general case, it was intended to enlarge the renormalizability condition. But still no one was able to prove or disprove the availability of fixed points in realistic 4D gauge theories such as QED.

By the mid-1970s, some progress in the constructive QFT occured that sheded light on the structure of nonrenormalizable theories and conditions under which these theories acquire the mathematical sense. In particular, it was found that some nonperturbative solutions of such theories can be specified by a finite number of arbitrary parameters as opposed to the situation in the perturbation theory (for a review of this progress see [42]). At present, the idea to weaken the renormalizability requirement and extend the class of physically admissible theories has lost its acuteness, and yet it does not consign to oblivion [43].

Another line of the renormalization group investigations culminated in the proof of the 'decoupling theorem' according to which, in a renormalizable theory with a field of mass M much greater than masses of other fields, it is possible to find such renormalization prescription that the heavy field decouples and reveal itself only as corrections to the Lagrangian suppressed by powers E/M where E is an energy characteristic to this region (for detail see chapter 8 of the book [8]). An important consequence is that low-energy physics is described by effective theories containing only those particles which are actually significant in the energy range under consideration. Thus the mass M plays the role of ultraviolet cutoff in the effective theory; if $E \to M$ then the effective theory fails to be applicable and should be replaced by a new effective theory with greater cutoff. For $E \ll M$, the heavy particles can reveal themselves only through the processes (for example, weak decays) which are forbidden by symmetries (for example, by the parity conservation) in the absence of the heavy particles (for example, W bosons) which mediate the interaction with the broken symmetry. These small effects correspond to nonrenormalizable terms of the Lagrangian since they are multiplied by inverse powers of M, and hence have the operator dimension of mass to a positive power. "Thus the only interactions that we can detect at ordinary energies are those that are renormalizable in the usual sense, plus any nonrenormalizable interactions that produce effects which although tiny, are somehow exotic enough to be seen" [11]. For example, the nonrenormalizable

four-fermion interaction $\frac{G}{\sqrt{2}}\,J\cdot J$ is suppressed by the smallness of the Fermi constant G (proportional to M_W^{-2}) for $E \ll M_W$, and yet observable due to the chiralness of weak processes.

What is the explanation of the renormalizability of the Standard Model in the context of effective theories? The answer is rather evident. The Standard Model is an effective low-energy theory that might be derived from a higher-level theory, e. g., from the Grand Unification, if one would integrate out all the heavy fields, e. g., X bosons, in the path integral. Since we are keeping in mind the observed world, the Lagrangian that governs it should not be suppressed by powers of $1/M_X$, whence, for dimensional reasons, it follows that it is renormalizable.

With this in mind, the gravitation Lagrangian, nonrenormalizable by the power-counting, should be suppressed by powers of $1/M_{\rm P} = \sqrt{k}$. However, it is well known that such is not the case: The Hilbert Lagrangian $\sqrt{-g}R/16\pi k$ is not divided but, quite the reverse, multiplied by $M_{\rm P}^2$. Although the Einsteinian gravity emerges as the low-energy limit of the string theory, it bears no relation to effective theories.

So, in the framework of the effective theory ideology, we found only part of the answer (not entirely convincing) to the above question. Moreover, we remained ignorant of the *dynamical* reason by which nonrenormalizable interactions are in disrepute or at least are suppressed by small coefficients. It is even misunderstood whether the renormalizability is an antipode of the nonrenormalizability.

Below we will discuss just these aspects of the renormalizability problem. Attention will be centered on the quantum physics at distances very short by macroscopical standards but much greater than the Planck length $l_{\rm P}$. We assume gravity as a quantum phenomenon to be of little importance in this region; its impact amounts to the classical curvature of spacetime. In Sec. 4 we will elucidate that the condition of renormalizability is equivalent to the condition of suppressibility of collapse. But we will approach this conclusion deliberately step by step. In Sec. 2 we will be concerned with topological types of evolution of point particles which enables us to look at the collapse from topological point of view. (One often claims that classical theory has nothing to do with the renormalizability problem altogether since there are no processes of the creation and annihilation of particles in it, and, thus, coupling constants do not subject to the renormalization. It is widely believed that the classical self-energy δm diverges differently than the quantum self-energy Σ. This seems to rule out any linkage between ultraviolet diseases in classical and quantum theories. Distinguished as those diseases may be, the criterion of viability of both theories is the same, the preventability of collapse. Furthermore, In Sec. 6 we will see that the powers of divergences of corresponding quantities in dual classical and quantum pictures are in effect coincide. Thus the analysis of ultraviolet properties of classical pictures is of great benefit to gaining insight into what happens in much less clear quantum pictures.) The simplest variant of the collapse, the fall to the centre, discussed in Sec. 3 allows to formulate the criterion of preventability of collapse in a general form: The collapse is preventable if the spectrum of the Hamiltonian is bounded from below. In Sec. 5 we will consider the similarity and difference of the concepts of renormalizability, kinetic dominance, and suppresibility of collapse. In Sec. 6 we will use the holographic principle to explain the origin of anomalies and make precise the relation between the renormalizability and reversibility. Section 7 sums up our discussion.

2 Topological types of evolution

There are two points of view on what is the basic object in the classical field theory. One of them takes fields as fundamental, and particles as special manifestations of the fields. The other takes particles fundamental while fields play the role of codes conveying information of the particle behavior. Feynman and Wheeler showed that both standpoints are on an equal footing in classical electrodynamics (see, e. g., [44]). Besides, it is well known that the quantization leads to a synthetic object, the quantized field.

It is convenient to begin with the classical picture turning to notions that are rather intuitive but very deep, the topological notions. We adopt the point of view by which particles are fundamental, and consider their evolution.

The only topological guide relevant to such a picture is the criterion of *compactness*. It discriminates between finite and infinite motions. In other words, all systems of particles can be classed as bound and

unbound. Thus, decays and recombinations are topologically significant events, while elastic scatterings are not.

A diagrammatic sketch of elementary topological types of evolution is displayed in Figure 1. For simplicity, world lines of only two particles are diagrammed, but one keeps in mind systems composed of any number of particles. A history of an unbound system executing an infinite motion is shown by diagram *1*. A history of a stable bound system is shown by diagram *2*; the motions of such systems are

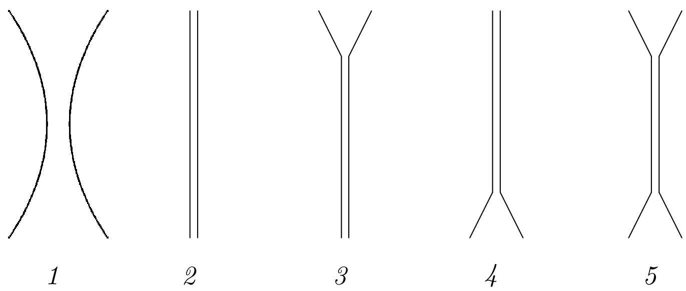

Figure 1: Elementary topological types of evolution

compactly supported. For some forms of interactions, a bound system can exist for a half-infinite period, whereupon it decays into separate fragments, as is shown by diagram *3*. Here we see a topological change of the regime of motion: From compact to noncompact. One might also conceive the formation of a bound system existing indefinitely long (there is a finite probability that nothing happens with it at a later time). The situation is shown by diagram *4*. Here we encounter the opposite regime change: From noncompact to compact. If the formed system decays after a lapse of a finite period, we arrive at the situation shown by diagram *5*[5]. Here we see a double topological change of the regime of motion: From noncompact to compact, and vice versa. The collection of variants of evolution *1*–*5* almost exhausts all the possible topological types. Histories of classical systems of point particles are built out of these elementary variants.

To take an example, imagine a realm where all particles are contained in clusters incapable of decay into individual particles, but capable of exchanging their constituents at collisions. It is just the situation of the cold subnuclear realm where quarks are confined in hadrons, and do not exist in the isolated form. The bulk of the hadron phenomenology is grasped by planar diagrams [45], which implies in particular that the individuality of valence quarks remains unchanged as long as they reside in an undisturbed hadron. Such persistence of valence quarks is peculiar to classical particles. That is why this realm can be described semiclassically. The planar diagrams are largely built out of the diagrams *1*–*5*. All one need to add to this set is the seagull-type diagram (two half-infinite timelike curves springing up from a single point). This diagram expresses the creation or the annihilation of a quark-antiquark pair. In the classical theory, the seagull-shaped world line configurations are forbidden due to their incompatibility with the least action principle. Creations and annihilations of valence quarks inside hadrons are actually suppressed by the Okubo–Zweig–Iizuka rule (see, e. g., [46, 47]), yet a moderate amount of such processes is tolerated at hadron collisions.

From the variants *1*–*5*, it is possible to separate subvariants corresponding to special states where the region of finite motion shrinks to a single point (we recall that the set containing a single point is compact). These special variants are displayed as diagrams similar to diagrams *1*–*5*, but with replacing two vertical lines by a single one, Figure 2, and are numbered by hatted numbers.

The special variant of evolution $\widehat{4}$ may (or may not) be identified with the collapse. When such is the case, the special variants differ significantly from the ordinary ones. Indeed, should the formation of an ordinary stable bound system be allowable (the variant *4*), in view of reversibility, decays of it are

[5]Diagram *5* turns to diagram *1* as lifetime of the formed system tends to zero.

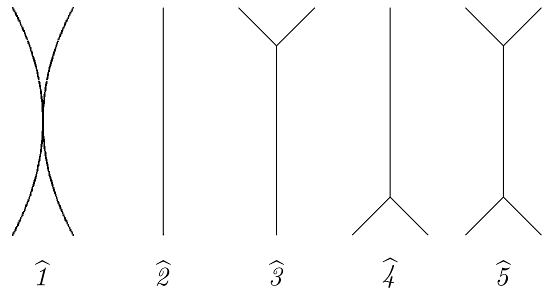

Figure 2: Special variants of evolution

also feasible (the variants *3* and *5*). On the other hand, as will be shown in the next section, a kind of collapse, the fall to the centre, is unavoidable, if the attraction potential is more singular than the centrifugal term. In this case, two particles merge into a single one, remaining amalgamated forever. Thus, the mere presence of the variant $\widehat{4}$ renders the inverse variant $\widehat{3}$ impossible (topologically, these diagrams can be distinguished by the number of arrows flowing in and flowing out the vertex), and also the variants $\widehat{1}$ and $\widehat{5}$. This suggests a topological violation of reversibility. The picture may be qualified as $\widehat{1}$-, $\widehat{3}$-, and $\widehat{5}$-deficient.

When attraction is less singular than centrifugal effects, the fall to the centre is prevented, and the variant $\widehat{4}$ disappears. The variants $\widehat{1}$, $\widehat{2}$, $\widehat{3}$ and $\widehat{5}$ are also missing from the picture due to lack of the reason for the formation of merged particles. We thus arrive at a depleted picture where all the variants $\widehat{1}$–$\widehat{5}$ are absent, but the reversibility is regained. We will see further that the separation of field theories into renormalizable and nonrenormalizable follows from the condition for suppressibility of collapse, which ensures the physical consistency of a given theory at the cost of the depletion of the topological picture, that is, at the cost of disappearance of all the special variants.

The outlined classification may be in a one-to-one way translated into the *spectral* language[6]. Indeed, the variants *1* and *2* correspond, respectively, to continuous and discrete parts of spectrum. The variant *3* implies the conversion of discrete spectrum of $|\,\mathrm{in}\rangle$ states to continuous spectrum of $|\,\mathrm{out}\rangle$ states, the variant *4* refers to the opposite conversion. The variant *5* is associated with a resonance line. It turns to the variant *1* as life time of the bound state vanishes, and the corresponding resonance line spreads to the extent that it ceases to be interpreted as an element of the discrete spectrum.

A less trivial illustration of this correspondence is that the aforesaid grasping the hadronic realm by planar diagrams can supposedly be expressed [48] in terms of Regge trajectories displayed as straight lines with a fixed slope on the Chew–Frautschi plot of hadronic mass squared m^2 versus spin J, with hadrons on any trajectory being separated by intervals $\Delta J = 2$. However, no convincing explanation of the correspondence between the planar diagram picture and the Regge equidistant spectrum was still proposed.

The special variants correspond to continuous spectra with a gap between the vacuum and one-particle energy levels (which is larger than the gap that was prior to the particle merger). If the variant $\widehat{4}$ is identifiable as the collapse, the depleted picture (without all the special variants $\widehat{1}$–$\widehat{5}$) corresponds to a spectrum bounded from below, whereas the $\widehat{1}$-, $\widehat{3}$- and $\widehat{5}$-deficient picture bears on systems with the energy spectrum unlimited from below.

3 The fall to the centre

As a prelude to the discussion of the collapse in systems with infinite number of degrees of freedom, we recall some aspects of the relativistic Kepler problem. This is a two-particle problem reducible to the

[6]Notice, this language is versatile enough to get rid of reference to the initial particle picture.

problem of a single particle moving in a potential $U(r)$ and specified by the Hamiltonian (see, e. g., [49], Sec. 39)

$$H=\sqrt{m^2+\frac{p_\phi^2}{r^2}+p_r^2}+U(r) \tag{2}$$

where p_ϕ and p_r are the momenta canonically conjugate to the polar coordinates ϕ and r. Note that p_ϕ is a conserved quantity, the orbital momentum J. Switching off the dynamics, i. e., taking $p_r=0$ in (2), one obtains the effective potential $\mathcal{U}(r)$ whereby the particle behavior near the origin is conveniently examined,

$$\mathcal{U}(r)=\sqrt{m^2+\frac{J^2}{r^2}}+U(r). \tag{3}$$

There are three alternatives. First, the attractive potential $U(r)$ is more singular than the centrifugal term J/r. Figure 3a depicts the effective potential $\mathcal{U}(r)$. The particle could, in principle, orbit in a circle of the radius corresponding to $\mathcal{U}_0$, the local maximum of $\mathcal{U}(r)$. But this orbiting is unstable, and the fall to the centre is highly probable, not to mention the case $E>\mathcal{U}_0$ when the fall to the centre is unavoidable.

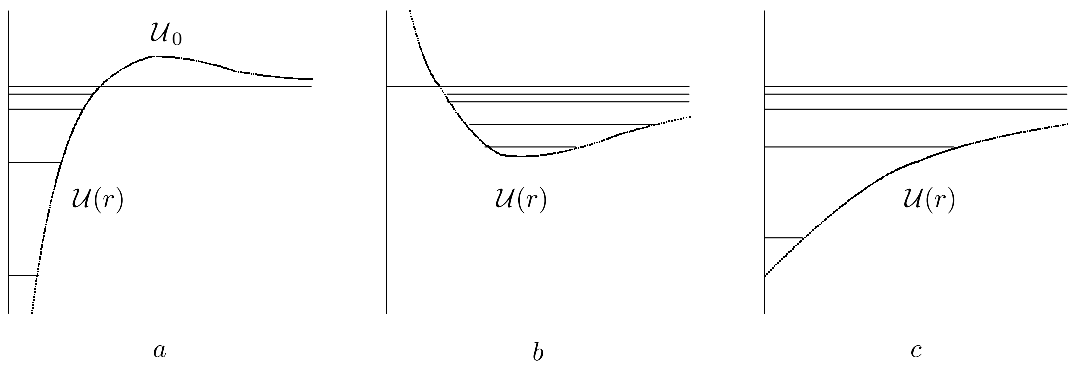

Figure 3: Effective potential and spectrum

Second, $U(r)$ is less singular than J/r. Specifically, for the potential $U(r)=-\alpha/r$ this means that $\alpha<J$. The shape of the $\mathcal{U}(r)$ curve is shown in Figure 3b. The particle executes a stable finite motion. The fall to the centre is impossible, except when $J=0$ which is associated with head-on collisions. But such collisions of point particles have zero probability measure if dimensionality of the collision arena is greater than 1.

Third, the singularities of $U(r)$ and J/r are identical, i. e., $\alpha=J$. The effective potential $\mathcal{U}(r)$ is exhibited in Figure 3c. The particle travels in a stable orbit that passes through the origin, but this does not arrest the movement.

The quantum-mechanical treatment [50] essentially confirms these conclusions. It follows from the solutions of relativistic wave equations for spin 0 and 1/2 particles that, in the case of sufficiently singular potentials $U(r)$, bound states form a discrete spectrum[7] extending from $E=m$ to $E=-\infty$; at $E=m$ there is a point of accumulation (Figure 3a). The system will tend to more and more advantageous states associated with successively lower energy levels. As this take place, the dispersion of the wave function

[7]Surprisingly, the merge of two particles into a single one does not result in a continuous spectrum signifying that the formed particle has no internal structure and free. In quantum mechanics, the individuality of objects does not preserve, and hence a superposition of states of two separate particles and the aggregate of two merged particles is possible. A particle itself is not fundamental, rather its degrees of freedom, in particular the energy spectrum, and environmental features, e. g., the potential $U(r)$ are fundamental concepts there.

diminishes to zero when $E_n \to -\infty$. The process closely resembles the fall to the centre in its classical interpretation. If $U(r)$ is less singular than the properly defined quantum-mechanical centrifugal term, the situation is conventional. The spectrum is bounded from below (Figure 3b). The only distinctive feature of the quantum-mechanical situation is that there exists a stable ground state with $J = 0$. However, this does not amount to the fall to the centre since the wave function behaves smoothly in the vicinity of the origin; there is balance between attraction and zero-point motion.

If singularities of $U(r)$ and the centrifugal term are identical, the approach based on the one-particle wave equations becomes invalid. At short distances, processes of the pair creation and annihilation enter the scene, and methods of QFT are called for. As the coupling α tends to its critical value α_c, a highly nontrivial picture can arise which continues to capture the theorists' attention (see, e. g., [51] and references therein).

As to the qualitative results of this analysis, they are quite reliable to suggest a criterion that separates systems where the fall to the centre is suppressible from those where it is inevitable. The criterion reads: The fall to the centre is preventable if and only if the energy spectrum is bounded from below.

4 Suppressibility of collapse

For systems with infinite number of degrees of freedom, the criterion should be extended to include them: *The tendency to collapse is suppressible if and only if the spectrum is limited from below.* Indeed, a negative contribution to the Hamiltonian is associated with attraction, and, on the other hand, attractive forces result in the collapse if the energy spectrum extends to minus infinity.

However, the spectrum of much of studied systems can hardly be conceived beforehand. The nonlinearity of dynamics can cause rearrangements of vacuum, whence it follows that naive realizations of the spectrum suggested by the formal structure of Hamiltonian is far from true.

We thus have to change *the statement of the problem*. We can confidently work with Hamiltonians quadratic in fields which in every case of physical interest possess spectra bounded from below. We split the Hamiltonian of the model at hand $H = \int d^{D-1}x\, \mathcal{H}(x)$ into two parts $H = H_0 + H_I$ where H_0 contains terms no greater than quadratic in fields, H_I containing the rest. H_0 is usually interpreted as the quantity governing the free field evolution, H_I being that responsible for the interaction. As is well known, in the free field case, the vacuum and one-particle states are stable, that is, vacuum fluctuations have no impact on the spectrum. If one requires that the vacuum root-mean-square fluctuation of H_0 be larger than that of H_I, then the spectrum structure, in particular the boundedness from below, will be left intact even in the presence of the interaction. Thus the tendency to collapse is suppressed in such systems in which

$$\Delta H_0 > \Delta H_I. \tag{4}$$

The analogy with the Kepler problem is fairly close: The reason for keeping the particle from the fall to the centre is that kinetic energy manifestations (centrifugal effects or zero-point motion) dominate over attractive forces.

Let us verify that (4) is equivalent to the power-counting criterion of the renormalizability. Because field operators in H are assumed to be normally ordered, the vacuum expectations $\langle 0|H_0|0\rangle$ and $\langle 0|H_I|0\rangle$ are vanishing, and (4) acquires the form

$$\langle 0|\, H_0^2|0\rangle > \langle 0|\, H_I^2|0\rangle. \tag{5}$$

(Note, this inequality may be regarded as a necessary condition for expanding in a perturbation series: For terms of the perturbation series to decrease from order to order, the 'unperturbed Hamiltonian' H_0 should exceed the 'perturbation' H_I in some sense.)

Both sides of (5) imply spatial integrations which generally result in ultraviolet divergences. The situation can be remedied in the usual fashion by introducing a point-splitting. Furthermore, on account of the positive definiteness of the integrands, it would be valid to compare not the results of the integrations but only singularities of the matrix elements $\langle 0|\, \mathcal{H}_0(x)\, \mathcal{H}_0(y)|0\rangle$ and $\langle 0|\, \mathcal{H}_I(x)\, \mathcal{H}_I(y)|0\rangle$ as $x^\mu \to y^\mu$.

The divergences emerging in this limit is evidence that the field quanta behave singularly on the light cone. But this is not quite the issue of our prime interest; a key role in the problem of collapse play short

Euclidean distances rather than short pseudo-Euclidean intervals. The ultraviolet divergences originate from short Euclidean distances, if the time variable x^0 is changed to ix^0. To justify this change one employs the analytic continuation which is possible provided that the matrix elements are regularized in such a way that the rotation of the time axis $x^0 \to ix^0$ does not intersect their singularities. Such is the case, if the regularization includes not only the point-splitting but also a chronological ordering of operator-valued fields.

The chronologically ordered multiplication is not the unique operation. There are actually two alternative definitions of the T-product, by Dyson and by Wick. One can adopt the Wick T_w-product (which is more convenient being commutative with differentiations), accompanying this choice by the replacement of Hamiltonian with Lagrangian [5, 52]. Then, instead of (5), we get

$$\langle 0|\, T_w\{\mathcal{L}_0(x)\, \mathcal{L}_0(y)\}|0\rangle > \langle 0|\, T_w\{\mathcal{L}_I(x)\, \mathcal{L}_I(y)\}|0\rangle \tag{6}$$

where $\mathcal{L}_0$ and $\mathcal{L}_I$ are the Lagrangian densities related, respectively, to $\mathcal{H}_0$ and $\mathcal{H}_I$. Dealing with the situation in which free-field fluctuations dominate over those of interaction, it is allowable to use the interaction picture where $\mathcal{L}_0$ and $\mathcal{L}_I$ depend of free fields.

Let the system be located in a flat D-dimensional spacetime and specified by a set of N real-valued fields generally symbolized as $\chi_j(x)$, $j = 1, \ldots, N$. (For now, let us ignore subtleties related to constrained systems [53]. They can reveal themselves in the final result only implicitly through presence or lack of the gauge invariance.) We set

$$\mathcal{L}_0 = \sum_{j=1}^{N} {:}\chi_j(x)\, Ł_j(\partial)\, \chi_j(x){:} \tag{7}$$

where $Ł_j(\partial)$ is a differential operator of the first order for fermions and of the second order for bosons, the symbol : : stands for the normal product. Let us consider the simplest nontrivial Lagrangian of interaction in the form

$$\mathcal{L}_I = g \prod_{i=1}^{n} {:}P^{k_i}(\partial)\, \chi_i(x){:} \tag{8}$$

where $P^{k_i}(\partial)$ is a differential operator of k_ith order.

Substitution of (7) and (8) into (6) yields expressions built out of Feynman propagators $\Delta_{Fj}(x)$ analytically continued to the D-dimensional Euclidean space. $\Delta_{Fj}(x)$ satisfies the equation

$$Ł_j(\partial)\, \Delta_{Fj}(x) = -\delta^D(x),$$

and can be recast in the form

$$\Delta_{Fj}(x) = Q^{r_j}(\partial)\, \Delta_F(x).$$

Here, $Q^{r_j}(\partial)$ is a differential operator of r_jth order specific to the field of the given spin, $\Delta_F(x)$ is a kernel of the operator $(\Delta + m_j^2)^{-1}$, and Δ is the D-dimensional Laplacian. Of concern to us is the situation at short Euclidean distances $x^2 = \epsilon^2$ where $\Delta_F(x) \sim x^{2-D}$, and

$$\Delta_{Fj}(x) \sim Q^{r_j}(\partial)\, x^{2-D} = O(\epsilon^{\,2-D-r_j}).$$

For estimating the formal quantity $\delta^D(0)$ one should introduce the ultraviolet cutoff $\Lambda \sim 1/\epsilon$ in the Fourier integral

$$\delta^D(x) = \frac{1}{(2\pi)^D} \int d^D k\, e^{ikx},$$

then one obtains

$$\delta^D(0) = O(\epsilon^{-D}).$$

With these estimations, in the limit $(x-y)^2 = \epsilon^{\,2} \to 0$, we have

$$\langle 0|\, T_w\{\mathcal{L}_0(x)\, \mathcal{L}_0(y)\}|0\rangle = \sum_{j=1}^{N} [Ł_j(\partial)\, \Delta_{Fj}(x-y)]^2 = O(\epsilon^{-2D}), \tag{9}$$

$$\langle 0|\, T_w\{\mathcal{L}_I(x)\,\mathcal{L}_I(y)\}|0\rangle = g^2 \prod_{i=1}^{n} P^{k_i}(\partial_x)\, P^{k_i}(\partial_y)\, \Delta_{Fi}(x-y) = O(\prod_{i=1}^{n} \epsilon^{\,2-D-r_i-2k_i}). \tag{10}$$

Comparing powers of ϵ in (9) and (10), we find that the condition (6) is met when

$$\Omega_n \,=\, D + \sum_{i=1}^{n} (1 - \tfrac{1}{2}\, D - \tfrac{1}{2}\, r_i - k_i) \geq 0. \tag{11}$$

One can readily see that the inequality (11) is nothing but the power-counting criterion for the renormalizability. For example, taking into account that $r_i = 0$ for scalar fields and $r_i = 1$ for Dirac fields, for $D = 4$, one finds $\Omega_n = -\omega_n$, where ω_n is defined by Eq.(1). Note also that the propagator of massive vector fields $(\eta_{\mu\nu} - \partial_\mu\partial_\nu/m^2)\,\Delta_F$ behaves as x^{-D} for short distances, as compared with the propagator of massless gauge fields $(\eta_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2)\,\Delta_F$, that behaves as x^{2-D}, whence it follows that the criterion of suppressibility of collapse (11) responds to presence or lack of the gauge invariance through the value of r_i specific to the vector fields in question.

If (11) is an exact equality, the coupling g becomes dimensionless. As this take place, the meeting of the condition (6) is feasible when the magnitude of g is sufficiently small.

The presented analysis must be considered as heuristic arguments of the preventability of collapse. The basis for more rigorous treatment might be, for example, properties of exact solutions to the Bethe–Salpeter equation that describes quantum-field bound states. Unfortunately, efforts of attacking this challenging problem (see, e. g., [54]) met with little success for lack of adequate mathematical technique.

5 Underlying reason for renormalizability

According to Dyson, a theory is renormalizable if all the ultraviolet divergences are absorbed by a redefinition of parameters in the Lagrangian. To fulfil this condition, it is necessary that vacuum fluctuations of the kinetic energy exceed those of the interaction. Thus the renormalizability can be substituted with the equivalent but more intuitive concept of the *kinetic dominance*[8].

It is a long-standing belief (see, e. g., [56]) that, in renormalizable theories, the physics at large distances is insensitive to the influence of short distances and may be effectively allowed for by a finite number of parameters, whereas such influence in the nonrenormalizable case is given by infinite number of parameters. Our analysis supports this idea. To be specific we turn back to the Kepler problem. In the situation where the fall to the centre is suppressed (Figure 3b), the run of the $\mathcal{U}(r)$ curve in the vicinity of the origin approaches to that of the I/r curve, hence the probe of short distances is controlled by the only parameter I. By contrast, in the situation where the fall to the centre is inevitable (Figure 3a), the run of the $\mathcal{U}(r)$ curve becomes close to that of the $U(r)$ curve as $r \to 0$, and, for the probe of short distances, account must be taken of infinite number of coefficients of the Laurent series representing $U(r)$.

Thus the renormalizability guarantees the *self-sufficiency of laws driving large-scale phenomena*. However, it is not to be supposed that all the phenomena occurring at the short-wave range are trivial, or at least can be unified with those in the long-wave range. Due to the suppression of collapse, the low-energy region under study is in effect isolated from the unexplored high-energy region.

It is well to bear in mind that the genuine condition of the preventability of collapse is not the kinetic dominance but the boundedness of the spectrum from below. Is this substitution of one condition for the other troublesome?

The kinetic dominance is *necessary* for the collapse to be suppressed[9], but *not sufficient*. Indeed, the Yukawa term $\mathcal{L}_I = ig\bar{\psi}\gamma_5\psi\phi$ in itself exemplifies a nonrenormalizable Lagrangian for $D = 4$ even though the condition (11) is met; the renormalizability cannot be reached unless the scalar self-interaction $\mathcal{L}_I = -\lambda\phi^4$ is added. Another well-known example is a theory of the Yang–Mills–Higgs type which is in general nonrenormalizable due to the presence of the axial anomaly. The anomaly violates the gauge

[8]D. I. Blokhintsev [55] was the first to suggest this concept in somewhat simplified and implicit form.

[9]Curiously enough, the role of the energodominance in the problem of gravitational singularity is exactly opposite: Breaking the energodominance prevents the singularity formation [57].

invariance making worse the ultraviolet behavior of the Yang–Mills fields, similar to that of massive vector fields. These examples demonstrate the existence of systems in which ultraviolet divergences defy their absorption, and hence the collapse is not suppressed despite the fulfillment of the inequality (4).

Arguing unsophisticatedly, one might even question the necessity of the kinetic dominance. For one thing, the impact of the apparently positive definite disturbance $\mathcal{H}_I = \lambda\phi^{2n}$, $\lambda > 0$, on the spectrum is at first glance quite harmless: The undisturbed energy levels seem to remain positive for any $n > 1$. However, an important fact is overlooked that the coupling λ can be profoundly altered in response to the vacuum polarization, in particular it can be opposite in sign to the bare coupling. Such a strong vacuum polarization is missing from two-dimensional superrenormalizable theories [58], so that n may be arbitrary for $D = 2$. As to the case $D = 4$, the restriction $n \leq 2$ implied by the condition (11) proves relevant here. Another doubt can be put by a statement known as the 'equivalence theorem' by which two quantum field theories related by a nonlinear field transformation

$$\chi = \xi + \xi^2 F(\xi) \tag{12}$$

have the same S matrix. It follows that the collapse is suppressed not only for polynomial Lagrangians satisfying the criterion (11), but also for any others obtained from them by the nonlinear transformations (12). However, using the technology of nilpotent operators, as it is applied in gauge field theories, one can show that the effect on $\mathcal{L}$ generated by the nonlinear part of this transformation is similar to a 'gauge fixing term' [59]. Thus, the condition (4) is in fact necessary for the suppression of collapse.

On the other hand, it may appear that the scalar self-interaction ϕ^3 delivers a counter-example of the 4D renormalizable theory with the Hamiltonian spectrum unlimited from below. Indeed, the term ϕ^3 tends to $-\infty$ more rapidly than the term $\frac{1}{2}m^2\phi^2$ tends to ∞ as $\phi \to -\infty$. In this consideration, however, the essential fact is overlooked; the leading term in the kinetic energy is not $\frac{1}{2}m^2\phi^2$ but $\frac{1}{2}(\partial\phi)^2$ and the singularity of the latter, $O(\epsilon^{-4})$, is comparable with the singularity of ϕ^4 clearly exceding the singularity of ϕ^3.

The kinetic dominance criterion pertains to systems disposed to collapse. Ultraviolet divergences warn of such disposition. The tendency to collapse is suppressed in some systems while it is accomplished without obstacles in the rest systems. One can judge which alternative is inherent in the given system not only from analytical but also from topological features of its behavior. The topological picture where the fall to the centre is the case is qualified as $\widehat{\mathit{1}}$-, $\widehat{\mathit{3}}$-, and $\widehat{\mathit{5}}$-deficient. This observation in the conventional local QFT can immediately be extended to more general theories (for example, M theory) as follows. Let conversions of some spatially extended objects to another ones of lesser dimension (e. g., contracting a string to a point, squeezing a membrane to a string, flatting a soliton to a plane wave, etc.) be possible on the classical level. Such conversions are reminiscent of the special variant of evolution $\widehat{\mathit{4}}$ where two initial particles are interpreted as two ends of contracting open string. If these conversions are invertible then every special variant of evolution is available, otherwise the picture is deficient: The very existence of some special variant is incompatible with the existence of the reverse variant. We take this deficiency to be *topological definition of the feasibility of collapse*. Simply stated, this deficiency is equivalent to an incurable failure of the time reversal. In the quantum mechanics language, this deficiency implies that the Hilbert spaces H_{in} and $\mathrm{H}_{\mathrm{out}}$ are different, that is, the unitarity is violated. As is shown in [60], the scalar theory with the Lagrangian of interaction ϕ^4 is trivial for $D > 4$. This hints that nonrenormalizable theories are unitary only in that the collapse is referred to 'pre-historic' times, and the relationship $\mathrm{H}_{\mathrm{in}} = \mathrm{H}_{\mathrm{out}}$ is assured for such part of the system which survive the collapse occuring in the remote past and in the following does not experience interaction.

The suppressibility of collapse is a remedy regaining the reversibility. But the cost for this is further depletion of the topological picture: Every special variant disappears. The remedy may be such vigorous that no realistic renormalizable theory with nontrivial S matrix is available. Although examples of nontrivial theories satisfying all the Wightman axioms in 2D and 3D spacetimes was shown to exist [58], the possibility that, say, the scalar self-interaction ϕ^4 in four-dimensional spacetime, an important part of the Standard Model, corresponds to $S = 1$ must not be excluded. This brings up the question: Is this depleted topological picture an immanence of each renormalizable theory?

6 Holographic principle and anomalies

It is likely that the reader's attention was drawn on the fact that the *spacetime dimension* D repeatedly come up in our discussion. For example, D enters the criterion of suppressibility of collapse (11). Further still $D = 2$ and $D = 3$ flashed in relation to the marvellous realm where the vacuum polarization is weak and the description is superrenormalizable. The very notion of the collapse proposed in the preceding section is based on conversions of extended objects to objects of lesser dimension. (It is intringuing that, in supersymmetric theories where divergences are partially or completely cancelled, the ultraviolet happiness is due to effective reductions of D. Indeed, as was shown by G. Parisi and N. Sourlas [61], a supersymmetric theory on the graded manifold with ordinary coordinates $x_1, \ldots, x_D$ and the Grassmannian coordinates θ and $\bar{\theta}$ is equivalent to a nonsupersymmetric theory on the manifold with the coordinates $x_1, \ldots, x_{D-2}$. In other words, the explicit realization of the supersymmetry calls for coordinates of *negative* dimensions such as θ and $\bar{\theta}$. Thus the supersymmetry is an elevator transporting us into lower dimensions.)

Another important to our theme notion is the *reversibility*. Up till now, these two notions were disconnected. We would gain considerable insight into the subject, if we would received at our disposal some device enabling a linkage of physical pictures for different D. Such a device actually exists. This is the so-called holographic principle. It allows to regard the irreversibility as an anomaly that spoils the dualism between *quantum* and *classical* descriptions of realms of adjacent dimensions.

The holographic principle was first proclaimed by 't Hooft [62] and L. Susskind [63] in the context of black holes. According to this principle, information on degrees of freedom inside a volume can be projected onto a surface (also called screen) which encloses this volume. For lack of the generally accepted formulation, we will turn to provisional versions of this principle discussed in the literature. Their essence is as follows: A classical theory (which includes gravity), describing phenomena within a volume, can be formulated as a quantum theory of these phenomena (without gravity) projected onto the boundary of this volume. In such a form, the holographic principle was confirmed in Ref.[64] where the consistency between semiclassical supergravity in an anti-de Sitter space and quantum superconformal Yang–Mills theory on the boundary of this space was revealed.

There is reason to believe that the holographic principle is valid also when gravity is excluded. Then it can be simply realized through a remarkable dualism due to V. de Alfaro, S. Fubini and G. Furlan [65]. They argued that the generating functional for Green's functions of Euclidean QFT in D dimensions coincides with the Gibbs average for classical statistical mechanics in $D + 1$ dimensions. In other words, there exists a correspondence (the AFF dualism) between the classical picture in a spacetime M_{D+1} and the quantum picture in D-dimensional sections of M_{D+1} at any instants. Such sections play the role of D-dimensional screens carrying the hologram of what happens in M_{D+1}.

We recall the idea of AFF dualism by the example of a system described by the scalar field $\phi(x)$. Let the system be located in a D-dimensional Euclidean spacetime and specified by the Lagrangian $\mathcal{L}$. One introduces a fictitious time t. The field becomes a function of the Euclidean coordinates $x_1, \ldots, x_D$ and fictitious time t, $\phi = \phi(x, t)$. If $\frac{1}{2}(\partial\phi/\partial t)^2$ is treated as the kinetic term, and $\mathcal{L}$ the potential energy term, then one defines a new Lagrangian

$$\tilde{\mathcal{L}} = \tfrac{1}{2}(\partial\phi/\partial t)^2 - \mathcal{L}$$

generating the evolution in t. The associated Hamiltonian is

$$\tilde{\mathcal{H}} = \tfrac{1}{2}\pi^2 + \mathcal{L}$$

where $\pi = \partial\tilde{\mathcal{L}}/\partial\dot{\phi} = \partial\phi/\partial t$ stands for the conjugate momentum which is assumed to obey the classical Poisson bracket

$$\{\phi(x, t), \pi(y, t)\} = \delta^D(x - y). \tag{13}$$

It is easy to see that the Gibbs average for an ensemble with the temperature $kT = \hbar$

$$\mathcal{Z}[J] = \int \mathcal{D}\pi\mathcal{D}\phi \, \exp\Big(-\frac{1}{kT}\int d^D x\, (\tilde{\mathcal{H}} + J\phi)\Big) \tag{14}$$

turns to the generating functional for the quantum Green functions

$$Z[J] = \int \mathcal{D}\phi \, \exp\Big(-\frac{1}{\hbar}\int d^D x\, (\mathcal{L} + J\phi)\Big) \tag{15}$$

upon taking the Gaussian integral over π. Note that the holographic mapping of the bulk picture in M_{D+1} onto the screen picture in a section of M_{D+1} at any instant t is ensured by the Liouville theorem. Indeed, although $\phi(x,t)$ and $\pi(x,t)$ evolve in t, the elementary volume in phase space $\mathcal{D}\pi\mathcal{D}\phi$, and with it the Gibbs average $\mathcal{Z}$ are t-independent. (For the AFF dual description of gauge systems see [65].)

Thus, it is meaningless to ask whether a given realm is classical or quantum. It may appear both as classical and quantum, but these two looks pertain to spacetimes of nearby dimensions[10]. To identify the realm, one should only indicate D. For example, assigning $D = 4$ to the electromagnetic realm, we bear in mind that it can be grasped by either some 'quantum' 4D Lagrangian $\mathcal{L}$ or the associated 'classical' 5D Lagrangian $\tilde{\mathcal{L}}$.

It is clear that the conventional procedure of quantization only shifts the seat to another realm: In lieu of the initial classical system living in D dimensions, a new classical system living in $D+1$ dimensions emerges. As is well known, symmetries of classical Lagrangians may be sensitive to the dimension; some of them are feasible only for a single $D^\star$, while another only for $D = 2n$. On the other hand, given a quantized $D^\star$-dimensional theory, we deal actually with the holographic image of classical $D^\star + 1$-dimensional theory, and the symmetry under examination is sure to be missing from it. Therein lies the reason for the symmetry damage due to quantization known as the 'quantum anomaly'. The responsibility for this damage rests with the fact that the compared classical descriptions merely differ in dimensions.

In the general case, the AFF dualism does not imply that the classical $D+1$-dimensional theory is *equivalent* to its quantum D-dimensional counterpart. There are two reasons for the lack of equivalence. First, the field theory can suffer from ultraviolet divergences, and the equality $\mathcal{Z} = Z$, strictly speaking, does not have mathematical sense. The dissimilarity of classical divergences from quantum ones keeps the expressions $\mathcal{Z}$ and Z from being equal. This lack of equivalence prohibits derivation of exact expressions for anomalies by a direct comparison of two classical actions of adjacent dimensions. Furthermore, it gives rise to a *classical anomaly* related to the violation of reversibility on the classical level with preserving this symmetry on the quantum level.

Second, given a one-to-one holographic mapping of a classical picture onto a quantum one, one finds that the image displays a violation of the classical determinism[11] implying the loss of information on the classical system behavior.

Let us briefly run through these issues following essentially Ref. [66].

6.1 Conformal anomaly

The holography is particularly suited for studying the origin of anomalies since one may repeatedly handle classical actions alone. Canonical transformations leave the Poisson bracket (13) invariant, hence the measure of integration in (14) $\mathcal{D}\pi\mathcal{D}\phi$ has a corresponding canonical invariance property enabling one to perform canonical transformations without having to introduce a Jacobian. This can be exemplified by the conformal anomaly in the Yang–Mills theory.

Let us consider the conformal transformation of the metric

$$g_{\mu\nu} \to e^{2\varepsilon}\, g_{\mu\nu} \tag{16}$$

resulting in the associated Noether current, the energy-momentum tensor

$$\Theta^{\mu\nu} = \frac{2}{\sqrt{-g}}\,\frac{\delta}{\delta g_{\mu\nu}}\,\sqrt{-g}\,\mathcal{L}. \tag{17}$$

The conformal invariance of the classical action S is attained if $\delta S = 2\varepsilon\, g_{\mu\nu}\Theta^{\mu\nu} = 0$, that is,

$$\Theta^{\mu}{}_{\mu} = 0. \tag{18}$$

[10]By comparison, 't Hooft in his recent work [67] claimed: "In our theory, quantum states are not the primary degrees of freedom. The primary degrees of freedom are deterministic states."

[11]Mathematically, the determinism is embodied in the requirement that the solution to any Cauchy problem for classical dynamical equations must be unique.

It follows from (17) that, in the classical $D+1$-dimensional Yang–Mills theory with

$$\mathcal{L}_{\rm YM} = -\frac{1}{4\Omega_{D-1}\alpha}\,{\rm tr}\,F_{\alpha\beta}F^{\alpha\beta},$$

$\Theta^{\mu\nu}$ is written as

$$\Theta^{\mu\nu} = \frac{1}{\Omega_{D-1}\alpha}\,{\rm tr}\,(F^{\mu\alpha}F_{\alpha}^{\ \nu} + \frac{1}{4}\,\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}). \tag{19}$$

Here, Ω_{D-1} is the area of a $D-1$-dimensional unite sphere, and α the Yang–Mills coupling. The expression (19) makes it clear that the condition (18) is fulfilled only for $D+1=4$. Yang–Mills equations are conformally invariant only in a four-dimensional spacetime. This is in agreement with the scale invariance of the theory manifested in the fact that the coupling α is dimensionless for $D+1=4$.

With the holographic principle, the quantization of the classical 4D Yang–Mills theory culminates in the classical 5D Yang–Mills theory where one finds $\Theta^{\mu}_{\ \mu} \neq 0$. This is how the conformal anomaly occurs.

Technically, the conformal invariance breakdown in QFT is traced to the mass scale μ introduced for the normalization of α. S. Coleman and E. Weinberg [68] called this the dimensional transmutation. Such a transmutation is missing from the quantum 3D Yang–Mills theory, thereby the conformal invariance of its AFF dual, the classical 4D Yang–Mills theory, is ensured. Why does this happen? In the next section, we will see that the quantum 3D theory is super-renormalizable, the vacuum polarization is weak, there is no infinite charge renormalization, and the need for μ disappears.

As to the quantum 4D theory, vacuum polarization effects are significant here, and α becomes the running coupling constant $\alpha(q^2/\mu^2)$ dependent on the momentum transfer squared q^2, implying that

$$\Theta^{\mu}_{\ \mu} = \frac{2g^{\mu\nu}}{\alpha^2}\,\frac{\partial\alpha}{\partial g^{\mu\nu}}\,\frac{1}{16\pi}\,{\rm tr}\,F^2 = \frac{1}{8\pi}\,\frac{\beta(\alpha)}{\alpha^2}\,{\rm tr}\,F^2 \tag{20}$$

where $\beta = \partial\alpha/\partial\log q^2$ is the Gell-Mann – Low function. We encounter the q^2-dependent coefficient of ${\rm tr}\,F^2$ as opposed to the naïve expression

$$\Theta^{\mu}_{\ \mu} = \frac{1}{8\pi^2\alpha}\,{\rm tr}\,F^2 \tag{21}$$

which might be derived from the classical 5D Yang–Mills action. Thus the holographic principle qualitatively explains the relation $\Theta^{\mu}_{\ \mu} \propto {\rm tr}\,F^2$, but the exact coefficient of the expression (20) does not reproduce.

6.2 Irreversibility

The violation of the AFF equivalence after the infinite renormalization can be regarded as a kind of anomalies. As an example, let us consider the emergence of the irreversibility in a classical picture, with the reversibility in the dual quantum picture being left intact. To be specific, take a D-dimensional quantum realm described by the scalar electrodynamics Lagrangian

$$\mathcal{L} = (\partial^\mu + ig_0A^\mu)\phi\,(\partial_\mu - ig_0A_\mu)\bar\phi - m_0^2\phi\,\bar\phi - \frac{\lambda_0}{4}(\phi\,\bar\phi)^2 - \frac{1}{4\Omega_{D-2}}\,F_{\mu\nu}F^{\mu\nu}. \tag{22}$$

In the dual $D+1$-dimensional classical realm, the scalar field ϕ can be treated as a Lagrangian coordinate of a continuous medium that evolves in time $t = x_{D+1}$. However, such a model is inconvenient for analysis of ultraviolet properties, and we will discuss its discrete analogue, the system with very large number (strictly speaking, ∞^D) of charged point particles. Assume that the action for the classical particle interacting with electromagnetic field is of the usual form

$$S = -\int d\tau\,(m_0\sqrt{v\cdot v} + g\,v\cdot A) \tag{23}$$

where $v^\mu \equiv \dot z^\mu \equiv dz^\mu/d\tau$ is the $D+1$-velocity of the particle, and τ the proper time.

Features of dual AFF pairs corresponding to several values of D are summarized in Table 1. Let us begin with the line 'Ultraviolet behavior' indicating the dependence of observables on the cutoff Λ. The maximal power of Λ increases with D to give the progressive violation of the AFF dualism up to its complete failure above $D=4$.

TABLE 1. **Dualities in scalar electrodynamics**

<table>
<tr><td colspan="8">Spacetime dimension of the quantum picture</td></tr>
<tr><td colspan="2">$D=1$</td><td colspan="2">$D=2$</td><td colspan="2">$D=3$</td><td colspan="2">$D=4$</td></tr>
<tr><td colspan="8">Dual AFF pairs</td></tr>
<tr><td colspan="2">$1\mathrm{D}_{\mathrm{quant}} - 2\mathrm{D}_{\mathrm{class}}$</td><td colspan="2">$2\mathrm{D}_{\mathrm{quant}} - 3\mathrm{D}_{\mathrm{class}}$</td><td colspan="2">$3\mathrm{D}_{\mathrm{quant}} - 4\mathrm{D}_{\mathrm{class}}$</td><td colspan="2">$4\mathrm{D}_{\mathrm{quant}} - 5\mathrm{D}_{\mathrm{class}}$</td></tr>
<tr><td colspan="8">Ultraviolet behavior</td></tr>
<tr><td>Λ^0</td><td>Λ^0</td><td>$\log\Lambda$</td><td>$\log\Lambda$</td><td>Λ</td><td>Λ</td><td>Λ^2, $\log\Lambda$</td><td>Λ^2, $\log\Lambda$</td></tr>
<tr><td colspan="8">Renormalizability</td></tr>
<tr><td>finite</td><td>finite</td><td>super-ren’zable</td><td>ren’zable</td><td>super-ren’zable</td><td>ren’zable</td><td>ren’zable</td><td>non-ren’zable</td></tr>
<tr><td colspan="8">Reversibility</td></tr>
<tr><td>holds</td><td>holds</td><td>holds</td><td>holds</td><td>holds</td><td>weak violat</td><td>holds</td><td>topolog violat</td></tr>
</table>

Among $D+1$-dimensional classical quantities, the dependence on Λ is inherent in the energy-momentum vector[12] of electromagnetic field generated by a point particle,

$$P_\mu = \int d\sigma^\nu\, \Theta_{\mu\nu}, \tag{24}$$

where the integration is performed over a D-dimensional spacelike hypersurface. In the static case when electromagnetic field can be specified by the potential φ satisfying the Poisson equation

$$\Delta\varphi(\mathbf{x}) = -\Omega_{D-1}\,\rho(\mathbf{x}) \tag{25}$$

with

$$\rho(\mathbf{x}) = g\,\delta^D(\mathbf{x}), \tag{26}$$

one has

$$\varphi(\mathbf{x}) = g\begin{cases} |\,\mathbf{x}|^{2-D}, & D\ne 2,\\ \log|\,\mathbf{x}|, & D=2.\end{cases} \tag{27}$$

From (26) and (27) in combination with the static expression for the particle self-energy

$$\delta m = \frac{1}{2}\int d^D\mathbf{x}\,\rho(\mathbf{x})\,\varphi(\mathbf{x}) \;=\frac{1}{2}\,g^2\varphi(0), \tag{28}$$

one obtains leading Λ-dependences indicated in Table 1.

For small deviations from statics, Eq. (24) acquires the form

$$P_\mu = c_1\,v_\mu + c_2\,\dot v_\mu + c_3\,\ddot v_\mu + \dots \tag{29}$$

where the variables v_μ, $\dot v_\mu$, etc., relate to the point source of the field at a given instant. It is clear that $c_1 = \delta m$. From dimensional considerations, one finds also that $c_i/c_{i+1}\sim\Lambda$.

Let us turn to $5\mathrm{D}_{\mathrm{class}}$. The integration of Eq. (24) is carried out over a four-dimensional hypersurface, and, therefore, only even powers of Λ are nonzero:

$$c_1\sim\Lambda^2,\quad c_2=0,\quad c_3\sim\log\Lambda. \tag{30}$$

The Λ^2 term is absorbed by the mass renormalization, but there is no parameter in the action (23) suitable for the absorption of the logarithmic divergence, and the $5\mathrm{D}_{\mathrm{class}}$ theory turns out to be *nonrenormalizable.* To regain the renormalizability, to the action (23) must be added a term with higher derivatives [69] of the type

$$-\,\kappa_0\int d\tau\,\frac{1}{\sqrt{v\cdot v}}\left(\frac{d}{d\tau}\,\frac{v^\mu}{\sqrt{v\cdot v}}\right)^2. \tag{31}$$

As to the dual D-dimensional quantum quantities, we are directly concerned with the polarization operator $\Pi_{\mu\nu}$, the scalar self-energy Σ, and the scalar self-interaction Υ. The relevant one-loop diagrams

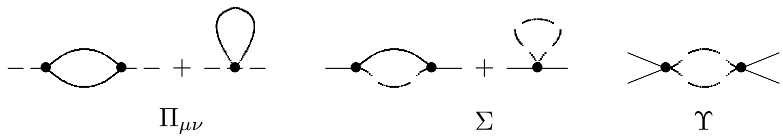

Figure 4: One-loop diagrams in scalar QED

are depicted in Figure 4. (The light-light scattering revealing a more soft ultraviolet behavior is immaterial here.) Using a gauge-invariant regularization, $\Pi_{\mu\nu}$ can be cast as $\Pi_{\mu\nu}(q) = (q^2\eta_{\mu\nu} - q_\mu q_\nu)\,\Pi(q^2)$. An elementary Feynman technique leads to

$$\Sigma \sim \int \frac{d^Dk}{k^2}, \qquad \Pi \sim \int \frac{d^Dk}{(k^2)^2}, \qquad \Upsilon \sim \int \frac{d^Dk}{(k^2)^2}.$$

It follows

$$\Sigma \sim \begin{cases} \Lambda^{D-2}, & D \neq 2, \\ \log\Lambda, & D = 2, \end{cases} \qquad \Pi \sim \begin{cases} \Lambda^{D-4}, & D \neq 4, \\ \log\Lambda, & D = 4, \end{cases} \qquad \Upsilon \sim \begin{cases} \Lambda^{D-4}, & D \neq 4, \\ \log\Lambda, & D = 4. \end{cases} \tag{32}$$

The comparison of (27)–(28) and (30) with (32) shows that the divergence powers in the $5\mathrm{D}_{\rm class}$ theory are the same as those in the $4\mathrm{D}_{\rm quant}$ theory. Nevertheless, the latter is *renormalizable* [70] since all the primitively divergent diagrams refer to the quantities Π, Σ and Υ which renormalize, respectively, g_0, m_0 and λ_0 in the Lagrangian (22).

Since the bar Lagrangian $\mathcal{L}_0$ and counter-terms in the $4\mathrm{D}_{\rm quant}$ theory have the same structure, and $\mathcal{L}_0$ is invariant under time reversal $t \to -t$, the renormalized quantum dynamics is reversible. Meanwhile the dual $5\mathrm{D}_{\rm class}$ theory is irreversible due to the non-suppressed tendency to collapse (unless the action is modified *ad hoc* by the addition of terms with higher derivatives). Indeed, in the Kepler problem, the potential energy $U(r) = -g^2/r^2$ is more singular than the centrifugal term J/r, and the fall to the centre is inevitable. The fall to the centre is an irreversible phenomenon. Note, the irreversibility has topological character: The mere presence of the variant of evolution $\widehat{4}$ (the fall to the centre) rules out the possibility of the inverse variant $\widehat{3}$ (splitting and subsequent departure of the merged particles). The violation of the AFF dualism $4\mathrm{D}_{\rm quant} - 5\mathrm{D}_{\rm clas}$ is due to the fact that the classical realm is devoid of the vacuum polarization which is responsible for infinite renormalization of g_0 and λ_0.

Let us turn to the case $D = 3$. Now Π and Υ are finite, hence the coupling constants are not subject to infinite renormalization. The divergences of both Σ and δm are absorbed by the mass renormalization. The number of parameters contained in the Lagrangians are sufficient for the removal of all divergences, so, at first glance, the AFF equivalence is the case. But this is wrong. The mass renormalization makes a finite *mark* which is more deep in the classical picture than in the quantum one. The renormalized quantum dynamics is reversible. By contrast, in the renormalized classical dynamics, the reversibility is violated. Indeed, the coefficients c_i in (29) are [71]

$$c_1 \sim \Lambda, \quad c_2 = -\frac{2}{3}\,g^2, \quad c_3 \sim \Lambda^{-1},$$

whence it follows the expression for the four-momentum of the ‘dressed’ particle

$$p_\mu = m\,v_\mu - \frac{2}{3}\,g^2\,\dot v_\mu \tag{33}$$

(m is the renormalized mass), and the equation of motion of the ‘dressed’ particle

$$m\,\dot v^\mu - \frac{2}{3}\,g^2\,(\ddot v^\mu + \dot v^2 v^\mu) = f^\mu. \tag{34}$$

[12] The angular momentum has the same ultraviolet behavior

This is the Lorentz–Dirac equation which is evidently not invariant under time reversal $\tau \to -\tau$. This irreversibility is due to the energy dissipation stemming from radiation of electromagnetic waves.

Many attempts to derive Eq. (34) from some Lagrangian have not met with success. It is likely that the partition function (14) whose construction is based on the Hamiltonian (which in turn implies the availability of the corresponding Lagrangian) has nothing to do with the renormalized classical dynamics. The irreversibility of the classical evolution in the four-dimensional realm is attributable not to ergodic properties but to the peculiarity of the self-interaction in classical electrodynamics that the field energy can only be emitted (not absorbed) that is, only dissipation (not cumulation) of energy is possible there. The irreversibility is an anomaly revealing itself in the violation of the AFF equivalence $3\mathrm{D}_{\mathrm{quant}}$ – $4\mathrm{D}_{\mathrm{clas}}$.

Thus a trait of the $4\mathrm{D}_{\mathrm{class}}$ theory is that the renormalization deliver it from the invariance under time reversal which would contradict to the macroscopic experience. It is significant that this excess invariance is not violated at the expense of non-invariant terms of the Lagrangian, since their presence would impair the invariance of the $3\mathrm{D}_{\mathrm{quant}}$ theory. The weak violation of the AFF equivalence $3\mathrm{D}_{\mathrm{quant}}$ – $4\mathrm{D}_{\mathrm{clas}}$ is adequate to the physical reality. It can be liken to the chiral anomaly making possible the decay $\pi^0 \to 2\gamma$ which is forbidden due to the excess symmetry of the Standard Model.

In the case $D = 2$, the ultraviolet situation is qualitatively the same as in the case $D = 3$: The divergences of both Σ and δm are absorbed by the mass renormalization. However, the renormalized $3\mathrm{D}_{\mathrm{class}}$ dynamics is reversible. Indeed, $c_1 \sim \log\Lambda$, therefore, $c_2 \sim \Lambda^{-1}$. This means that the mass renormalization makes no finite mark in the three-momentum of the 'dressed' particle governed by the second Newton law. Thus the AFF equivalence $2\mathrm{D}_{\mathrm{quant}}$ – $3\mathrm{D}_{\mathrm{clas}}$ is observed.

In the case $D = 1$, there are no ultraviolet divergences at all, and the AFF equivalence cannot be violated for this reason. This case should be discussed more elaborately.

6.3 Indeterminism in a classical realm

Ultraviolet divergences is not the only reason why the AFF dualism is not an exact equivalence. Another reason is that classical pictures are devoid of quantum coherence, in other words, the classical determinism cannot be reconciled with the quantum principle of superposition. The notion of probability is incorporated in classical statistical mechanics quite artificially; it expresses the measure of our ignorance of deterministic pictures in detail, whereas the probability amplitude is a fundamental element of quantum theory. The holographic mapping of a classical theory onto a quantum theory should suffer information loss, but the mechanism of this loss is obscure[13]. We will give a schematic reasoning demonstrating that this problem is tractable in the case $D = 1$, namely, a kind of indeterministic behavior of particles is possible in the two-dimensional classical realm.

Let us write the dynamical equations of the discussed $2\mathrm{D}_{\mathrm{class}}$ theory:

$$\partial_\lambda F^{\lambda\mu}(x) = 2e \sum_{a=1}^{2} \int_{-\infty}^{\infty} d\tau_a \, v_a^\mu(\tau_a) \, \delta^{(2)}\Big(x - z(\tau_a)\Big), \tag{35}$$

$$m_a \dot{v}_a^\mu = e_a v_\nu^a F^{\mu\nu}(z_a). \tag{36}$$

They are exactly integrable. From the form of solutions to these equations, two striking features of the $2\mathrm{D}_{\mathrm{class}}$ realm follow. First, there is no radiation of electromagnetic waves in this realm [69] and hence there is no dissipation of energy. All motions of particles are reversible. Second, it is possible that several point particles merge into a single aggregate, and then, after a lapse of some period, split into the initial objects.

For simplicity, we restrict our consideration to a system of two particles with equal masses m and charges e (denoting $e^2/m = a$) which are to be related to the centre-of-mass frame. Let the particles be moving towards each other, and their total energy is such that, at the instant of their meeting, their

[13]To circumvent this difficulty, 't Hooft [67] suggested that "Since, at a local level, information in these (classical) states is not preserved, the states combine into equivalence classes. By construction then, the information that distinguishes the different equivalence classes is absolutely preserved. Quantum states are equivalence classes."

velocities are exactly zero. Then there exists an exact solution to Eqs. (35) and (36) describing two worldlines $z_1^\mu(\tau)$ and $z_2^\mu(\tau)$ which coalesce at the instant τ^* and separate at the instant $\tau^{**}=\tau^*+T$,

$$z_1^\mu(\tau)=\begin{cases} a^{-1}\{\sinh a(\tau-\tau^*),1-\cosh a(\tau-\tau^*)\}, & \tau<\tau^*,\\ \{\tau-\tau^*,0\}, & \tau^*\le\tau<\tau^{**}, ,\\ a^{-1}\{aT+\sinh a(\tau-\tau^{**}),\cosh a(\tau-\tau^{**})-1\}, & \tau\ge\tau^{**}\end{cases} \tag{37}$$

$$z_2^\mu(\tau)=\begin{cases} a^{-1}\{\sinh a(\tau-\tau^*),\cosh a(\tau-\tau^*)-1\}, & \tau<\tau^*,\\ z_1^\mu(\tau), & \tau^*\le\tau<\tau^{**},\\ a^{-1}\{aT+\sinh a(\tau-\tau^{**}),1-\cosh a(\tau-\tau^{**})\}, & \tau\ge\tau^{**},\end{cases} \tag{38}$$

and the retarded field $F^{\mu\nu}$ expressed through $z_1^\mu(\tau)$ and $z_2^\mu(\tau)$ as

$$F^{\mu\nu}(x)=e\sum_{a=1}^{2}\frac{1}{\rho_a}\,(R_a^\mu v_a^\nu-R_a^\nu v_a^\mu). \tag{39}$$

Here, $R_a^\mu\equiv x^\mu-z^\mu_{a\,\rm ret}$ is an isotropic vector drawn from the emitting point $z^\mu_{a\,\rm ret}$ on ath worldline to the point of observation x^μ, and $\rho_a=R_a\cdot v_a$ the invariant retarded distance between x^μ and $z^\mu_{a\,\rm ret}$.

The parameters τ^* and τ^{**} are arbitrary. If τ^* and τ^{**} are different and finite, then the curves (37) and (38) correspond to the formation of an aggregate with finite life time, the variant $\widehat{5}$. For $\tau^{**}\to\infty$, they describe the formation of a stable aggregate never decaying, the variant $\widehat{4}$. For $\tau^*\to-\infty$, we see decay at a finite instant of an aggregate formed at the infinitely remote past, the variant $\widehat{3}$. If $\tau^*\to-\infty$ and $\tau^{**}\to\infty$, then the curves degenerate into a straight line corresponding to an absolutely stable aggregate, the variant $\widehat{2}$. For $\tau^*=\tau^{**}$, they describe an aggregate which exists for a single instant, the variant $\widehat{1}$. Thus the solution to Eqs. (35) and (36) with the given Cauchy data is not unique. Moreover, we have a continuum of solutions since the period of the merged state can be any $T\ge 0$. The decay occurs quite accidentally at any instant. Note, however, that variants of evolution with such Cauchy data comprise null set.

In summary, a $D+1$-dimensional classical picture can be equivalent to the AFF associated D-dimensional quantum picture if and only if $D=1$. Indeed, the $1\mathrm{D}_{\rm quant}$ theory is free of ultraviolet divergences. It is just the two-dimensional classical realm in which radiation is absent, the problem of dissipation of energy does not arise, and hence there is no anomaly of reversibility. Only in this realm, the retarded electromagnetic field is not singular on world lines of its sources, Eq.(39), and the special variant of evolution $\widehat{4}$ must not be identified with the fall to the centre. Besides, only this realm is compatible with the special variants of evolution rendering dynamics stochastic already on the classical level (in higher dimensions, special variants of evolution either comprise a deficient picture or disappear altogether). Since the measure of such variants in phase space is zero, they have no effect on the fulfillment of the Liouville theorem, so that the variable t is outside the quantum description where survives only ix_0 playing the role of the 'Euclidean' time. Taking into account that clusters of classical charged particles in the two-dimensional realm mimic classical strings, it becomes clear that the behavior of classical strings is coded in the behavior of quantum point objects.

7 Conclusion

We have endeavored to argue that the renormalizability amounts to the kinetic dominance. The latter is necessary for suppressing the tendency to collapse. Why do we worry about the collapse? The picture where the collapse occurs is irreversible, with the irreversibility being of topological origin. Such a picture is topologically deficient; in the quantum mechanical language, the spaces of asymptotic states $\mathrm{H}_{\rm in}$ and $\mathrm{H}_{\rm out}$ differ, so that the unitarity is flawed at least on the perturbative level.

We have clarified features of the collapse and criterions for its suppression by the example of the fall to the centre.To prevent the collapse it is necessary and sufficient that the energy spectrum be bounded from below.

Noteworthy also is that the energy in supersymmetric theories is always formally positive since the lowest eigenvalue of any supersymmetric Hamiltonian is zero. A supersymmetric system is certain to

be immune from the collapse if the ground state of this system is *stable*. On the other hand, the supersymmetry allows to cancel divergences and sometimes renders the theory finite. One can try to connect these facts assuming that the supersymmetry shifts the scene to a realm of lesser dimension where the tendency to collapse is reduced. The so-called theorems of non-renormalization in supersymmetric theories implicitly confirm this assumption. Indeed, the vanishing of quantum corrections to coupling constants in supersymmetric theories implies that the picture is semiclassical (the vacuum polarization is weak) which is feasible only for sufficiently low dimension.

Changing the requirement of the renormalizability to that of the suppressibility of collapse is a step towards developing a kind of 'QFT intuition'. L. D. Faddeev [72] is emphatic that this task is rather urgent. It is not unlikely that this step will bring us nearer answering the question: "Why are three fundamental interactions renormalizable, while the rest one nonrenormalizable?".

However, what is the *practical* utiliy of this change? The intuitive understanding of QFT allows to avoid many puzzles without resort to intricate calculations. For example, it is widely believed that the divergence of the self-energy in classical electrodynamics δm is stronger than that in QED Σ. This belief stems from the fact that the divergence of δm is linear for $D = 4$ while the respective divergence of Σ is logarithmic. This gave rise to speculations implying that the quantization can improve the ultraviolet behavior of the theory. But this is false. Firstly, properties of a classical spinless particle must be compared with the appropriate properties of the scalar (not spinor) quantum field, in which case the divergence of Σ is quadratic, whence it follows that the quantization only deteriorates the ultraviolet situation. Secondly, any $D+1$-dimensional classical field theory relates holographically to a D-dimensional QFT, and, as was shown in Sec.6.2, δm and Σ diverge uniformly in such dual theories.

Another long-standing myth was that the introduction of a lattice instead of the spacetime continuum automatically eliminates all the ultraviolet divergences. This myth was shattered in 1988 by the following counter-example [32]. Let derivatives of the Dirac and scalar fields in $\mathcal{L}_0$ be replaced by finite differences, and $\mathcal{L}_I$ be given as $\mathcal{L}_I = -\lambda\phi^3 + ig\bar{\psi}\psi\phi$, where arguments of all the fields are taken coincident. One-loop diagrams of such a lattice model turn out to be divergent, and the model itself is nonrenormalizable. Lattice theories can suffer from their own ultraviolet infinities!

Armed with the idea of the kinetic dominance, this conclusion would not have appeared so much sensational. Recall that any lattice theory is a special instance of continual theories with nonlocal form-factors. Indeed, given the finite differences $\chi(x + l\,\hat{n}) - \chi(x)$, the free Lagrangian $\mathcal{L}_0$ is smeared out by the nonlocal operators of the form $\exp(l\,\hat{n}\cdot\partial) - 1$. Such a smearing is spread over a finite region of size l. Then the estimate $O(\epsilon^{-2D})$ in (9) is substituted by $O(l^{-2D})$. But all the fields in $\mathcal{L}_I$ are localized in the same points, hence the singularity of the expression (10) remains intact. In other words, we are dealing with the model where singularities of the kinetic term have been smeared out while those of the interaction term have been preserved. Now the condition (6) does not hold, the collapse is inevitable, and the model proves nonrenormalizable.

A fundamental difference between renormalizable and nonrenormalizable theories is that they correspond to quite different topological pictures of evolution. The topological manifestation of the nonrenormalizability is that the mere presence of some special variant of evolution excludes the possibility of existence of the reverse variant. Renormalizable theories are free of such a skewness: The reversibility is saved at the cost of the depletion of topological pictures where all the special variants of evolution are absent altogether. However, the question as to whether this depletion can result in that every four-dimensional renormalizable theory is trivial (maybe in some nonperturbative sense) still remains to be solved.

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